Spatially correlated temperature fluctuations in turbulent convection

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By measuring the degree of spatial correlation in temperature fluctuations at two points separated by a distance perpendicular to the mean flow, we are able to determine the viscous boundary layer thickness in turbulent convection. We demonstrate this method using water as the working fluid and find excellent agreement with directly measured results. Furthermore, from the most probable delay time for a thermal disturbance to successively pass the two temperature probes, we deduce the width of the mixing zone and again find very good agreement between the value obtained and that predicted by theory.

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I. INTRODUCTION

One central focus in studies of turbulent convection is to determine how the efficiency of heat transport (i.e., the Nusselt number Nu) depends on the Rayleigh number Ra, which measures the relative strength of buoyancy to dissipation. More importantly, the main question has been: What is the real mechanism that dictates heat transport, or the Nu and Ra relationship, in the turbulent state of thermal convection? The discovery of the hard turbulence state is a major development in the studies of thermal convection [1]. A defining feature of hard turbulence has been the power law dependence $\mathrm{Nu}\sim\mathrm{Ra}^{\beta}$ with the exponent $\beta=2/7$. This result is now being challenged both experimentally and theoretically. Questions arise as to whether the value of β is 2/7 [2], or even whether there should be a single power law [3,4]. In our view, the verdict is still out at the moment. One thing is clear, however; these studies have made it more apparent that global measurements, such as Nu vs (Ra, Pr), are not enough for a full understanding of the turbulent convection problem. Rather, one needs to make quantitative measurements of the local properties of the temperature and velocity fields, such as boundary layer thickness and shear rate, and test some of the specific predictions and assumptions concerning local quantities made in the various theoretical models.

The thermal and viscous boundary layers in the convection cell play key roles in determining the efficiency of heat transport and the various scaling and statistical properties. The first experimental study that systematically measured the Ra dependence of the viscous boundary layer in the hard turbulence state was carried out by Belmonte, Tilgner, and Libchaber (BTL) [5]. Using an indirect technique—the coincidence between the peak position of the cutoff frequency (the highest excitation frequency above noise level) of the temperature power spectrum and that of the velocity in their respective profiles with distance from the boundary—these authors found that in pressurized SF_6 gas the thickness of the viscous boundary layer (δ_v) follows the thermal boundary layer for $10^6\leqslant\mathrm{Ra}\leqslant10^7$, remains more or less constant for $10^7\leqslant\mathrm{Ra}\leqslant10^9$, and scales as $\mathrm{Ra}^{-0.44}$ (which the authors approximate as $\mathrm{Ra}^{-0.5}$) for $\mathrm{Ra}\geqslant2\times10^9$ all the way to the highest Ra reached in the experiment ($\sim10^{11}$). In a later experiment using a direct light scattering technique [6], we found that in water $\delta_v\sim\mathrm{Ra}^{-0.16}$, which holds in four convection cells with a combined range of Ra spanning from 10^7 to 10^{11} [7,8]. So within a substantially overlapping range of Ra the two experiments give quite different results for δ_v, albeit for systems with different Prandtl numbers. Because the coincidence between the peak position of the cutoff frequency and that of the velocity was observed at a single Rayleigh number in water [5], when BTL applied it to a gas they made two implicit but important assumptions: (1) this coincidence is valid for other values of Ra, and (2) it holds for other Prandtl numbers ($\mathrm{Pr}\simeq7$ in water and $\simeq0.7$ in gas). Since this ''power spectra method'' was introduced, it has been used in many different fluids with Pr ranging from ~0.025 (mercury) to $\sim10^3$ (glycerol) [9,10], and without explicit experimental verification.

We have recently tested the power spectra method in water for a range of Rayleigh numbers that span almost two decades and found that the length scale determined by this method indeed agrees well with the directly measured boundary layer thickness. But our test was inconclusive because a major justification given for this method was not borne out in water [11]. Since this justification places specific requirements on the properties of thermal plumes, our result raises questions about the validity of generalizing the power spectra method to fluids of different Prandtl numbers.

Another motivation for developing a technique to determine the viscous boundary layer thickness in nonaqueous fluids is related to the search for the ultimate state in turbulent convection, which has attracted much attention in recent years [12]. As predicted by Kraichnan in the 1960s, at very high Rayleigh numbers turbulent convection will enter an asymptotic state in which $\mathrm{Nu}\propto\mathrm{Ra}^{1/2}$ [13]. Thus heat is transported more efficiently in this ultimate state than in the hard turbulence state where $\mathrm{Nu}\propto\mathrm{Ra}^{2/7}$. More recently, Shraiman and Siggia also suggested the existence of this asymptotic state in their model for turbulent convection [14]. Although the two models are based on somewhat different arguments, both place explicit requirements on the behavior of the viscous boundary layer. In an experiment conducted in mercury, Glazier *et al.* reported that this ultimate state does not exist [15], thus contradicting an earlier claim for its existence by Chavanne *et al.* [16]. One of the arguments used by Gla-

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zier *et al.* to dismiss the ultimate state is that their viscous boundary layer became thinner than the thermal one and yet they still observed $Nu \propto Ra^{2/7}$. However, their viscous boundary layer thickness was obtained by using the power spectra method [17]. Since mercury and helium are perhaps the most promising fluids with which to search for the asymptotic state, it is important that a viable technique suitable for these fluids be developed for determining the viscous boundary layer thickness. This would enable one to verify some of the specific predictions about the ultimate state, in addition to the important Nu measurements.

We present here a method for determining the viscous boundary thickness that utilizes the variation of the spatial correlations between temperature fluctuations at different locations in the convection cell. In addition, we also study the interplay between coherent thermal structures (plumes and thermals) and large-scale flow by investigating the behavior of the most probable time delay in the cross-correlation function of temperature fluctuations between two spatial points. From the profile of the delay time with height, we identify a length scale and associate it with the width of the mixing zone. The concept of a mixing zone was first proposed by Castaing *et al.* in their model for hard turbulent convection [18]. It is a key ingredient in that model and specific predictions for both its Ra dependence and its magnitude have been made [18,19]. But to the best of our knowledge no quantitative measurements have been done on the width of the mixing zone.

II. EXPERIMENT

The convection cell used in the experiment has been described in detail previously [20]; we mention here only its key features. As shown in Fig. 1, it is a vertical cylinder of height 19.6 cm and of inner diameter 19 cm. The top and bottom plates are made of copper and the sidewall is made of Plexiglas. The control parameter in the experiment is the Rayleigh number $Ra = \alpha g L^3 \Delta / \nu\kappa$, with g being the gravitational acceleration, L the height of the cell, and α, ν, and κ, respectively, the thermal expansion coefficient, the kinematic viscosity, and the thermal diffusivity of water. The varying range of Ra in the experiment was from 2×10^8 to 2×10^{10}. During our experiment, the average temperature of the water in the convection cell was kept near room temperature and only the temperature difference across the cell was changed. In this way, the variation of the Prandtl number $Pr = \nu/\kappa$ is kept at a minimum ($Pr \simeq 7$).

The local temperatures are measured using two thermistors 300 μm in size with a time constant of 10 ms [21]. The thermistors are mounted on a syringe needle of 0.5 mm in diameter which in turn is soldered on a 1 mm diameter stainless steel tube. The thermistors traverse together vertically along the central axis of the cell during measurement. Each of the thermistors and three other resistors form an ac Wheatstone bridge that is modulated by a sinusoidal signal of 1 kHz. The output of each bridge is fed to a lock-in amplifier for demodulation and then sent to a dynamical signal analyzer (HP35670A with four channels) for digitization and recording. The voltage time series obtained is first converted into a resistance and then a temperature series, using calibrated conversion curves. From the temperature-time series, the cross-correlation function between the two thermistors is computed.

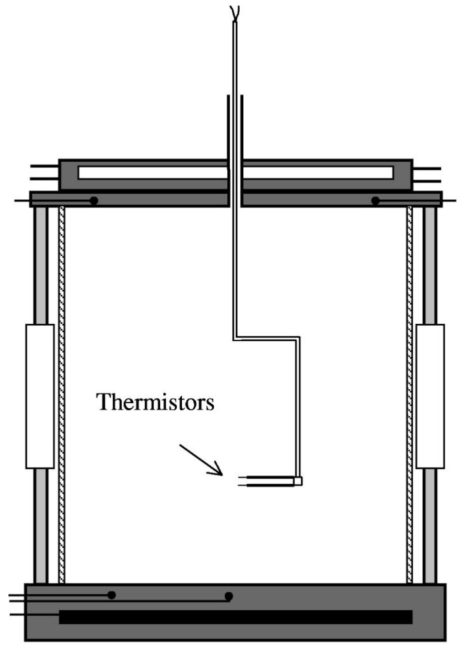

FIG. 1. Schematic diagram of the convection cell with two movable thermistors.

III. RESULTS AND DISCUSSION

A. Determination of the viscous boundary layer thickness

Cross-correlation flowmeters are commercially available industrial instruments that find wide applications in fluid speed measurements [22]. The basic principle of such instruments is simply to measure the time delay for a disturbance to successively pass two points *spaced along the flow direction*. With respect to turbulent convection, the two-temperature-probe cross-correlation technique has been applied by many groups to measure the velocity of the large-scale circulation near the *sidewall* of the convection cell [23,16,2]. Because of the presence of strong temperature fluctuations, however, the delay time and velocity are not related in any simple way near the conducting top and bottom plates. Thus the time-delay method cannot be used to obtain velocity near these regions.

In our method (which does not measure velocity), we measure the cross-correlation function of temperature fluctuations at two points *spaced perpendicular to the mean flow direction*. The principle of the technique is based on the following observations. Near the top and bottom conducting surfaces, the large-scale circulation acts like a horizontal wind that sweeps across the surface and creates the viscous boundary layer. Because of the presence of strong shear within the viscous layer, temperature fluctuations or inhomogeneities (such as coherent thermal objects) will have a much reduced probability of propagating straight upward and, therefore, of being detected at locations directly above where

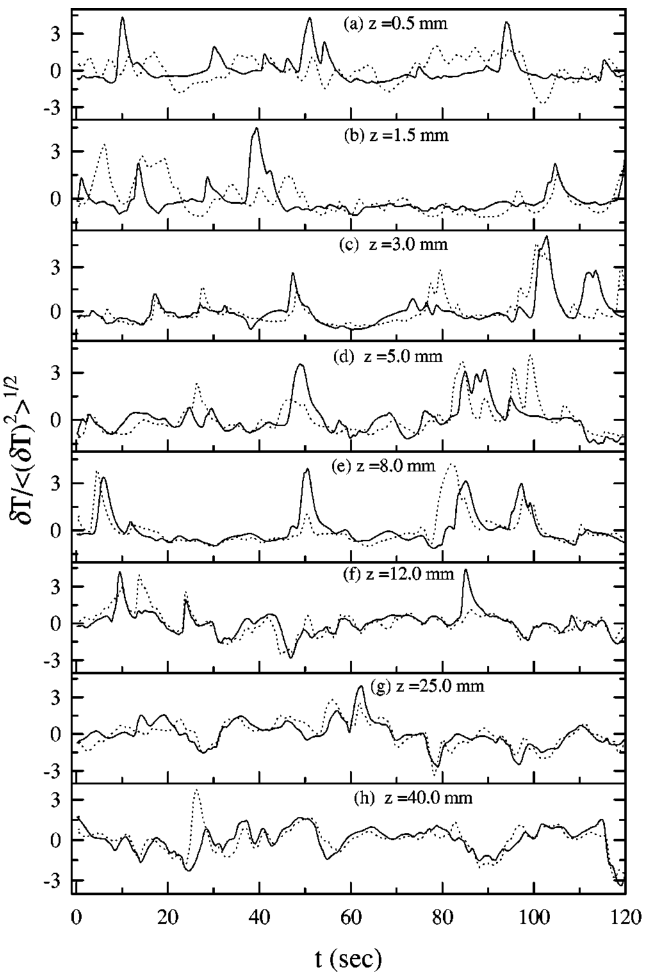

FIG. 2. Normalized temperature fluctuations recorded by two thermistors at different vertical positions with separation $\ell = 5.0$ mm and $\mathrm{Ra}=2.2\times10^9$. Solid line represents the upper probe temperature and dashed line that of the lower probe.

they originate. In other words, if we have a pair of temperature probes, one within the viscous layer and the other directly above the first one but outside the viscous layer, the correlation in temperature fluctuations at the two probe positions will be less than it would be if both were outside the viscous boundary layer where viscous shear (or horizontal velocity gradient) is much smaller. To test this idea, we conduct our experiment in water for which directly measured velocity boundary layer data are available for comparison.

Figure 2 shows time series of the normalized temperature fluctuations, $\delta T(t)/\langle(\delta T)^2\rangle^{1/2}$ $[\delta T(t)=T(t)-\langle T\rangle]$, measured by the two probes at various heights z from the lower probe to the bottom plate of the cell. The displayed time series are 2 min segments from 2 h long recordings, with probe separation $\ell=5.0$ mm and at $\mathrm{Ra}=2.2\times10^9$. These time series clearly show that the degree of correlation between temperature fluctuations at the two probe positions increases as they move away from the boundary. A measure of this degree of correlation is the cross-correlation coefficient $R(z)=C_z(\tau_0)$, where $C_z(\tau_0)$ is the amplitude (or maximum value) of the cross-correlation function

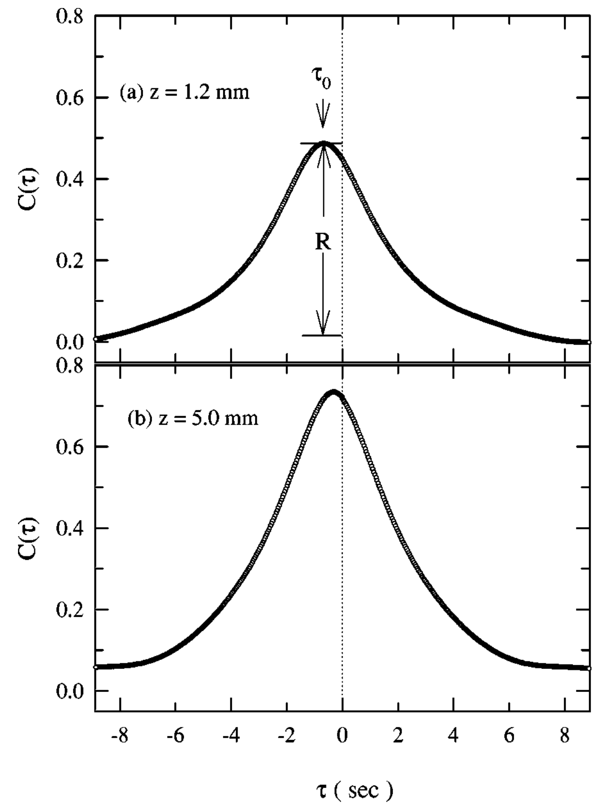

FIG. 3. The cross-correlation function of temperature fluctuations with $\ell=3.58$ mm and $\mathrm{Ra}=2.2\times10^9$, at (a) $z=1.2$ mm and (b) $z=5.0$ mm. R is the amplitude of the correlation function and τ_0 is the most probable delay time.

$$C_z(\tau)=\frac{\langle\delta T_1(t)\delta T_2(t+\tau)\rangle}{[\langle(\delta T_1)^2\rangle\langle(\delta T_2)^2\rangle]^{1/2}} \tag{1}$$

of the lower probe temperature $T_1(t)$ and that of the upper probe $T_2(t)$. Figure 3 shows two cross-correlation functions measured at $\mathrm{Ra}=2.2\times10^9$ with $z=$ (a) 1.2 mm and (b) 5.0 mm. It is clear from the figure that the amplitude R (indicated by the arrow) increases with increasing probe distance from the lower plate. Note also from the figure that the peak position τ_0, which represents the most probable delay time, is negative. This will be addressed in Sec. III B. Figure 4 shows a profile of the correlation coefficient $R(z)$ with the distance z, measured at $\mathrm{Ra}=7.1\times10^9$. It is seen clearly that $R(z)$ increases monotonically with z, and as the probes move further away from the surface R starts to level off. The ''turning point'' in the profile is an indication that the lower probe (T_1) has emerged from the shear layer and this position may be used as a measure of the viscous layer thickness. Outside the viscous boundary layer, the mean horizontal velocity decreases and eventually becomes zero at the cell center. But it does so gradually such that (at each height z) its variation over a distance of the order of the probe separation ℓ is small, and this is why the two-probe correlation retains its maximum value in this region.

It is also interesting to note that near the plate $R(z)$ has a linear dependence on z (inset of Fig. 4). This can be understood as follows. Inside the viscous boundary layer, the

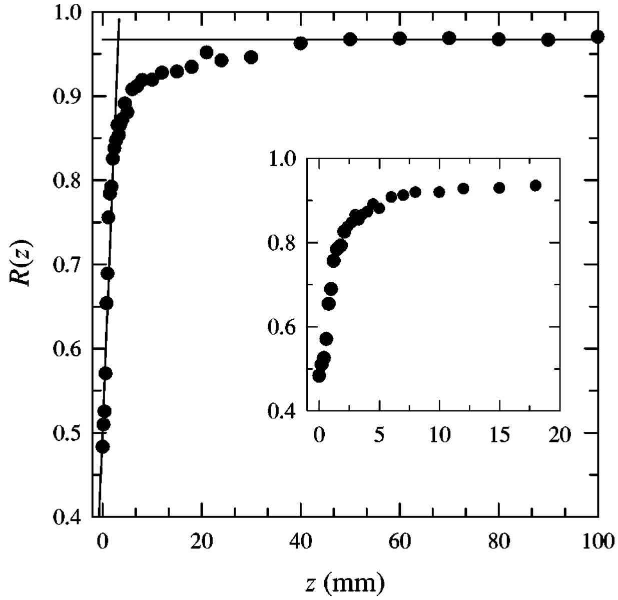

FIG. 4. The profile of R vs z at $\mathrm{Ra}=7.1\times10^9$. The inset shows an enlarged portion near the boundary.

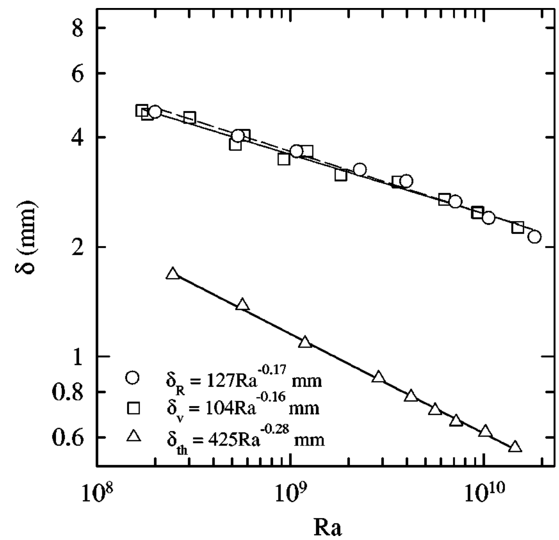

FIG. 5. Ra dependence of δ_R (circles) as obtained from the $R(z)$ profile and the viscous boundary layer thickness δ_v (squares) obtained by the light scattering technique. The thermal boundary layer δ_{th} (triangles) is shown for comparison.

horizontal velocity increases linearly with a large gradient (large shear rate), whereas outside it the velocity decays much more slowly (and can even be approximated as a constant over a distance of the order of the boundary layer thickness) [24,7]. Because one probe is inside and the other is outside the boundary layer, as the two traverse upward together the difference in horizontal velocities at the two probe positions decreases linearly. This leads to a linear increase in the degree of correlation in temperature fluctuations between the two positions. This feature prompted us to operationally define a length scale δ_R similar to the definition for the viscous boundary layer thickness δ_v using a true velocity profile [24]: we simply extrapolate the linear part of $R(z)$ near the surface and the horizontal line far away from the boundary, and denote the intersection point as δ_R. This procedure essentially allows one to determine the above-mentioned turning point in a consistent way. For the profile in Fig. 4, $\delta_R = 2.66$ mm, which is in excellent agreement with $\delta_v = 2.65$ mm measured by the light scattering technique [7]. We show in Fig. 5 the dependence of δ_R on Ra, along with that for δ_v from direct measurement. It is clear from the figure that, within the range of Ra spanned in the present cell, δ_R obtained using the two-thermistor method can be used as an alternative measure for the viscous boundary layer thickness. In Fig. 5 we also plot the thermal boundary layer thickness δ_{th} for comparison.

From the principle of the technique, it should be obvious that the separation ℓ of the two probes has no quantitative bearing on the measured δ_R as long as it is larger than the viscous layer thickness, and not too large so that the correlation of the two signals will still have a reasonable level of signal to noise ratio. To verify this, we varied ℓ between 1.7 and 7 mm and found that as ℓ is decreased the correlation between the two probes is increased so that $R(z)$ profiles such as the one shown in Fig. 4 become smoother (less data scattering), and that the best result is achieved when ℓ is just slightly larger than the viscous layer thickness. So this ℓ is taken as the optimum value. In fact, the various points in Fig. 5 were obtained with several values of ℓ, i.e., larger ℓ for lower Ra and smaller ℓ for higher Ra, but ℓ is fixed for the same Ra.

It is clear from the above discussion that our method requires only the existence of a mean flow that is dominant in the horizontal direction near the conducting surfaces, which is a well-established feature of high Rayleigh number convection. And no assumptions are made about the properties of the thermal plumes, which may behave differently under different Prandtl numbers [9]. Hence, this technique should apply to any fluid in the turbulent convection regime regardless of the value of the Prandtl number. It should be noted that this method is different in principle from most in-line cross-correlation flowmeters and does not measure the value of the velocity either directly or indirectly.

B. Measurement of the mixing zone

We discuss now the interaction between the thermal plumes and the large-scale circulation. As shown in Fig. 3, the peak position τ_0 of the correlation function is negative. τ_0 represents the most probable delay time for a temperature disturbance to pass the two probes in succession. According to the definition of the cross-correlation function $C(\tau)$, a negative τ_0 implies that most ''thermal objects'' passed the upper probe before they reached the lower one, even though our measurements were conducted near the bottom plate. This can be understood as follows. Because of the horizontal velocity gradient near the boundary, the top part of a thermal plume that is detached from the thermal boundary layer moves faster than the lower part. Therefore, the plumes are tilted while being advected downstream horizontally. In fact this has been observed previously in shadowgraph visualizations by Zocchi *et al.* [25]. We depict this in the cartoon in

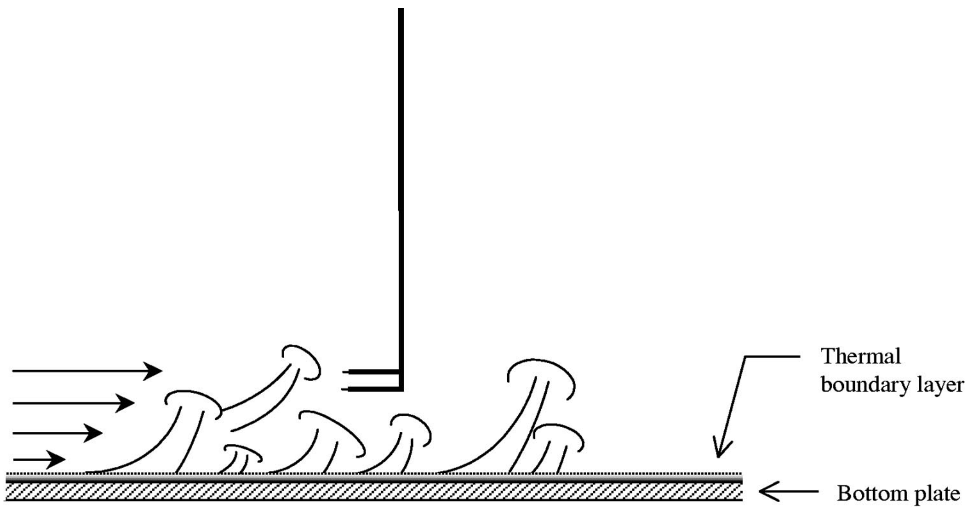

FIG. 6. A cartoon depicting the interaction between thermal plumes and the large-scale mean flow in the boundary layer region.

Fig. 6, where the tilted plumes are seen to reach the upper probe before the lower one and thus give rise to a negative delay time in the cross-correlation function. Because the mean flow is predominantly along the horizontal direction in the boundary layer region, the tilt of the plumes is most severe in this region, which means that τ_0 is most negative there.

In Fig. 7 we show a profile of τ_0 vs z at $\mathrm{Ra}=7.1\times10^9$. It is seen clearly from the figure that τ_0 is smallest (most negative) near the bottom plate, increases to about zero with increasing z, and then decreases again. At $\tau_0\simeq0$ the profile has a well-defined peak and this is found to be the case for all Rayleigh numbers measured. The behavior of τ_0 can be understood as follows. Near the boundary the vertical component of the velocity is negligible, so the average tilt of the plumes is largely determined by the horizontal velocity gradient. Outside the boundary layer, the vertical velocity starts to increase while the horizontal one decreases; this results in an increase in τ_0. Finally, the vertical velocity reaches its maximum value and starts to decay along with the horizontal one, at which point τ_0 reaches its maximum value ($\simeq0$). After τ_0 has reached its maximum value, it starts to decrease again, reflecting the fact that both the vertical and horizontal components are decreasing in this region but the horizontal one is always larger [26] (this is also the reason why τ_0 never becomes significantly positive).

The position of the maximum τ_0 defines a length scale δ_d. As δ_d corresponds approximately to the position of the maximum vertical velocity, it may be associated with the upper boundary of the mixing zone first proposed by Castaing *et al*. for the hard turbulence state [18]. In their original model, the authors argued that the mixing zone is where the (vertical) velocity of the fluid is accelerated to its value in the central region. The existence of a mixing region has been confirmed by visualization studies [25], but to our knowledge its width has not been quantitatively measured. If the length scale δ_d can be used to indicate the top boundary of the mixing zone, then we can define the width of the zone as $\delta_m=\delta_d-\delta_{th}$, where δ_{th} is the thermal boundary layer thickness. Figure 8 shows δ_m as a function of Ra (solid circles) together with the theoretical prediction $l_m=2L\times\mathrm{Ra}^{-1/7}$ by

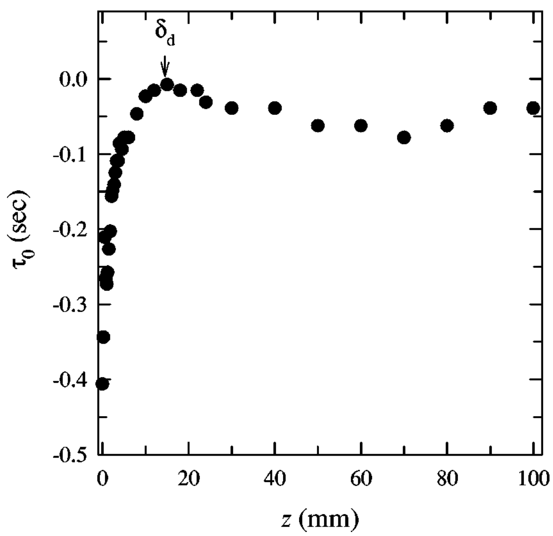

FIG. 7. The profile of τ_0 vs z at $\mathrm{Ra}=7.1\times10^9$.

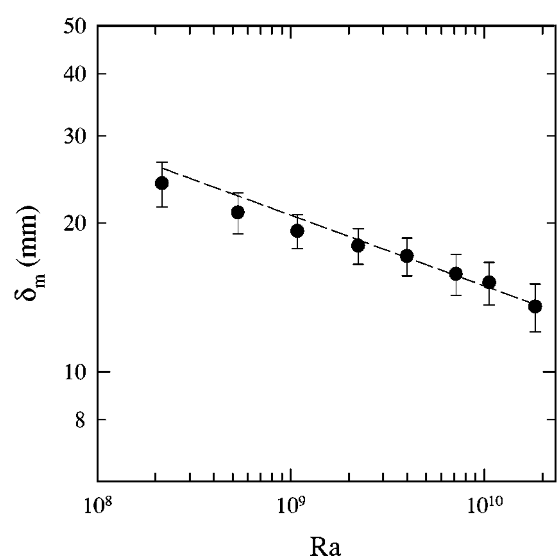

FIG. 8. δ_m as a function of Ra. The dashed line is the theoretical prediction $l_m=2L\times\mathrm{Ra}^{-1/7}$ for the width of the mixing zone.

Procaccia *et al.* [19]. It is remarkable that the two agree so well without any fitting parameter. Note also that δ_m is much larger than any boundary layer length scales measured in the convection cell (see Fig. 5, for example). We would like to caution, however, that, since the prefactor determined in Ref. [19] is for a fluid with $\mathrm{Pr}\simeq 1$ whereas in our experiment $\mathrm{Pr} \simeq 7$, the almost perfect agreement could be accidental (of course, it could also mean that the width of the mixing zone is not sensitive to Pr).

IV. CONCLUSION

By measuring the spatial correlation of temperature fluctuations between two thermistor probes spaced perpendicular to the flow direction, we are able to determine the viscous boundary layer thickness in a Rayleigh-Bénard convection cell in water. The results obtained are in excellent agreement with those measured by the direct light scattering technique. The technique takes advantage of the existence of a predominantly horizontal coherent mean flow near conducting plates and should be applicable to any fluid in the turbulent convection regime. At a minimum, it can serve as a cross-check to the power spectra method. Presently, experiments aimed at testing this method in other nonaqueous fluids with different Prandtl numbers are under way.

By studying the most probable delay time for a thermal disturbance to pass the two probes in succession, we obtain information about the interplay between coherent thermal structures (plumes and thermals) and the large-scale flow. We further identify a length scale from the profile of the delay time as measured by the correlation function. We associate this length with the width of the mixing zone and find excellent agreement with theoretical predictions.

Through the determination of the above two length scales, it is seen that much can be learned by studying the properties of spatial correlation in temperature fluctuations, which are manifestations of the interplay between coherent thermal objects and the large-scale circulation in the convection cell. With this contribution, we hope to stimulate more multipoint measurements to probe the spatial structures in turbulent convection.

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